Solving Quadratic Equations

Solving quadratic equations (equations with x^2 can be done in different ways. We will use three different methods in this packet. The first method is solving by factoring. For instance, if the equation was $x^2 - 22 = 9x$, you would have to subtract 9x from both sides of the equal sign so the equation would be $x^2 - 9x - 22 = 0$.

***Solve by factoring*:** After the equation is set equal to 0, you factor the trinomial.

$x^2 - 9x - 22 = 0$

$(x-11)(x+2) = 0$

Now you would set each factor equal to zero and solve. Think about it, if the product of the two binomials equals zero, well then one of the factors has to be zero.

$x^2 - 9x - 22 = 0$

$(x-11)(x+2) = 0$

$x - 11 = 0$ $\quad$ $x + 2 = 0$
$+11 \quad +11$ $\quad$ $-2 \quad -2$

$x = 11$ or $x = -2$
$x = \{-2, 11\}$ $\quad$ * Check in the ORIGINAL equation!

Solving each quadratic by factoring:

1) $x^2 - 5x - 14 = 0$ $\quad$ 2) $x^2 + 11x = -30$ $\quad$ 3) $x^2 - 45 = 4x$

4) $x^2 = 15x - 56$

5) $3x^2 + 9x = 54$

6) $x^3 = x^2 + 12x$

7) $25x^2 = 5x^3 + 30x$

8) $108x = 12x^2 + 216$

9) $3x^2 - 2x - 8 = 2x^2$

10) $10x^2 - 5x + 11 = 9x^2 + x + 83$

11) $4x^2 + 3x - 12 = 6x^2 - 7x - 60$

12) $6x^2 - 5x - 11 = 7x^2 - 5x - 47$

13) $4x^2 - 9x + 3 = 2x^2 + 5x + 3$

Q4 Quiz 1 Review

1) $x^2 = 22x - 96$

2) $5x^2 – 3x - 15 = 6x^2 + 10x + 15$

3) $8x^2 – 6x + 72 = 9x^2 - 6x – 72$

4) $5x = x^2 – 84$

5) $6x^2 = 30x + 396$

6) $x^3 + 39x = 16x^2$

7) $20x^2 - 45 = 0$

8) $3x^2 = -3x + 216$

9) $5x^3 - 210x = 5x^2$

10) $5x^2 + 35x = 40$

11) $21x^2 + 3x - 10 = 5x^2 + 3x - 9$

12) $x^2 = -22x - 72$

13) $11x^2 - 4x - 15 = 7x^2 - 4x - 14$

14) $4x^2 - 17x + 3 = 6x^2 - 9x + 3$

15) $8x^2 + 2x - 11 = 5x^2 + 9x - 11$

16) $3x^2 - 6x + 8 = 12x^2 + 21x + 8$

Answer Key:

1) x = {6,16} ***2) x = {-10, -3}*** ***3) x = {-12,12}*** ***4) x = {-7,12}***

5) x = {-6,11} ***6) x = {0,3,13}*** ***7) x = $\{-\frac{3}{2}, \frac{3}{2}\}$*** ***8) x = {-9,8}***

9) x = {-6,0,7} ***10) x = {-8,1}*** ***11) x = {- ¼ , ¼ }*** ***12) x = {-18,-4}***

13) x = {- ½ , ½ } ***14) x = {-2,0}*** ***15) x = $\{0, 7/3\}$*** ***16) x = {-3,0}***

Solve using the quadratic formula:

Solve $x^2 - 9x - 22 = 0$ using the *quadratic formula*

When $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a is the coefficient of x^2 **b** is the coefficient of x **c** is the number (third term)

Notice the ± is what will give your two answers (just like you had when solving by factoring)

$x^2 - 9x - 22 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 1
b= - 9
c = 22

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(22)}}{2(1)}$$

→ ***$(-9)^2$ -4(1)(22) can be done in one step in the calculator (leave out radical!!).***

$$x = \frac{9 \pm \sqrt{169}}{2}$$

Split and do the + side and - side

$$\frac{9 - 13}{2} \qquad \frac{9 + 13}{2}$$

x = -2 or x = 11

x = {-2,11}

**** Check in the ORIGINAL equation!***

Remember, ALL SUBSTITUTIONS must be done in PARENTHESES!!!!!!

Solving each quadratic using the Quadratic Formula:

14) $2x^2 - 6x + 1 = 0$

15) $3x^2 + 2x = 3$

16) $4x^2 + 2 = -7x$

17) $7x^2 = 3x + 2$

18) $3x^2 + 6 = 5x$

19) $9x - 3 = 4x^2$

20) $4x^2 - 5x = 2$

21) $11x^2 - 3 = -4x$

Answer Key:

14) $x = \frac{3}{2} \pm \frac{\sqrt{7}}{2}$

15) $x = \frac{-1}{3} \pm \frac{\sqrt{10}}{3}$

16) $x = \frac{-7}{8} \pm \frac{\sqrt{17}}{8}$

17) $x = \frac{3}{14} \pm \frac{\sqrt{65}}{14}$

18) No Real Solution

19) $x = \frac{9}{8} \pm \frac{\sqrt{33}}{8}$

20) $x = \frac{5}{8} \pm \frac{\sqrt{57}}{8}$

21) $x = \frac{-2}{11} \pm \frac{\sqrt{37}}{11}$

Q4 Quiz 2 Review

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

You must write the quadratic formula for each problem

1) $5x^2 - 3 = -4x$

2) $6x^2 = 12x - 5$

3) $2x^2 - 5x = 4$

4) $x^2 = -14x - 19$

5) $4x^2 = 6x + 3$

6) $7x^2 + x + 3 = 0$

7) $16x^2 + 1 = 12x$

8) $6x^2 = 7x + 1$

9) $12x^2 - 13x - 4 = 0$

10) $8x^2 + 16x + 3 = 0$

Answer Key:

1) $x = \frac{-2}{5} \pm \frac{\sqrt{19}}{5}$ ***2)*** $x = 1 \pm \frac{\sqrt{6}}{6}$ ***3)*** $x = \frac{5}{4} \pm \frac{\sqrt{57}}{4}$ ***4)*** $x = -7 \pm \sqrt{30}$

5) $x = \frac{3}{4} \pm \frac{\sqrt{21}}{4}$ ***6) No Real Solution*** $(\sqrt{-83})$ ***7)*** $x = \frac{3}{8} \pm \frac{\sqrt{5}}{8}$

8) $x = \frac{7}{12} \pm \frac{\sqrt{73}}{12}$ ***9)*** $x = \{-\frac{1}{4}, \frac{4}{3}\}$ ***10)*** $x = -1 \pm \frac{\sqrt{10}}{4}$

Factor:

1) $x^2 + 4x + 4$ 2) $x^2 - 6x + 9$ 3) $x^2 - 18x + 81$

4) $x^2 + 10x + 25$ 5) $x^2 - 20x + 100$ 6) $x^2 + 8x + 16$

7) $x^2 - 22x + 121$ 8) $x^2 + 32x + 256$ 9) $x^2 - 40x + 400$

Completing the Square

Completing the square is another method that is used to solve quadratic equations. This method is especially helpful when the quadratic equation cannot be solved by simply factoring.

Remember the standard form for a quadratic equation is: $ax^2 + bx + c = 0$.

Example:

1. $x^2 + 8x - 9 = 0$

$x^2 + 8x - 9 = 0$

$+9 \quad +9$

$x^2 + 8x = 9$

$\left(\frac{1}{2}(8)\right)^2 = (4)^2 = 16$

$x^2 + 8x + 16 = 9 + 16$

$x^2 + 8x + 16 = 25$

$(x + 4)(x + 4) = 25$

$(x + 4)^2 = 25$

$\sqrt{(x + 4)^2} = \sqrt{25}$

$x + 4 = \pm 5$

$x + 4 = 5$ $\qquad$ $x + 4 = -5$

$-4 \quad -4$ $\qquad$ $-4 \quad -4$

$x = 1$ $\qquad$ $x = -9$

$x = \{-9, 1\}$

Steps:

1. Be sure that the coefficient of the highest exponent is 1. If it is not divide each term by that value to create a leading coefficient of 1.
2. Move the constant term to the right hand side.
3. Prepare to add the needed value to create a perfect square trinomial. Be sure to balance the equation.
4. To create the perfect square trinomial:
   a) Take $\left(\frac{1}{2}b\right)^2$
   b) Add that value to both sides of the equation.
5. Factor the perfect square trinomial.
6. Rewrite the factors as a squared binomial.
7. Take the square root of both sides.
8. Split the solution into two equations
9. Solve for x.
10. Create your final answer.

More Examples:

1) $x^2 + 2x - 3 = 0$ 2) $x^2 - 16x + 60 = 0$ 3) $x^2 - 8x + 7 = 0$

Example:

1. $x^2 - 10x - 7 = 0$

$x^2 - 10x - 9 = 0$

$+7 \quad +7$

$x^2 - 10x = 7$

$\left(\frac{1}{2}(-10)\right)^2 = (-5)^2 = 25$

$x^2 + 10x + 25 = 7 + 25$

$x^2 + 10x + 25 = 32$

$(x - 5)(x - 5) = 32$

$(x - 5)^2 = 32$

$\sqrt{(x-5)^2} = \sqrt{32}$

$x - 5 = \pm 4\sqrt{2}$

$+5 \quad +5$

X = 5 $\pm 4\sqrt{2}$

Steps:

1. Be sure that the coefficient of the highest exponent is 1. If it is not divide each term by that value to create a leading coefficient of 1.

2. Move the constant term to the right hand side.

3. Prepare to add the needed value to create a perfect square trinomial. Be sure to balance the equation.

4. To create the perfect square trinomial:

   a) Take $\left(\frac{1}{2}b\right)^2$

   b) Add that value to both sides of the equation.

5. Factor the perfect square trinomial.

6. Rewrite the factors as a squared binomial.

7. Take the square root of both sides.

8. Isolate X. Since you cannot combine it with $\pm 4\sqrt{2}$, you do not need to split it into two equations.

9. Create your final answer

4) $x^2 + 12x + 4 = 0$

5) $x^2 - 8x - 11 = 0$

6) $x^2 + 6x - 36 = 0$

7) $x^2 + 4x - 44 = 0$

8) $x^2 + 24x + 24 = 0$

9) $x^2 - 16x - 6 = 0$

10) $x^2 + 6x + 4 = 0$

11) $x^2 - 10x + 5 = 0$

12) $x^2 + 2x - 47 = 0$

13) $x^2 + 4x - 92 = 0$

14) $x^2 - 28x + 4 = 0$

15) $x^2 - 12x - 20 = 0$

16) $x^2 + 18x + 1 = 0$

17) $x^2 - 22x - 39 = 0$

18) $x^2 + 8x - 164 = 0$

Q4 Quiz 3 Review

Solve each quadratic using completing the square:

1) $x^2 + 6x - 112 = 0$

2) $x^2 - 20x + 64 = 0$

3) $x^2 - 10x + 7 = 0$

4) $x^2 + 8x + 8 = 0$

5) $x^2 - 2x - 23 = 0$

6) $x^2 + 6x - 63 = 0$

7) $x^2 + 20x - 8 = 0$

8) $x^2 - 24x + 16 = 0$

9) $x^2 - 18x - 88 = 0$

10) $x^2 + 12x - 39 = 0$

11) $x^2 + 4x - 60 = 0$

12) $x^2 - 14x + 17 = 0$

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13) $x^2 - 10x + 7 = 0$

14) $x^2 + 8x + 15 = 0$

15) $x^2 - 2x - 80 = 0$

16) $x^2 + 6x - 39 = 0$

17) $x^2 + 20x - 25 = 0$

18) $x^2 - 24x + 23 = 0$

19) $x^2 - 18x - 40 = 0$

20) $x^2 + 12x - 18 = 0$

21) $x^2 + 15x + 26 = 0$

22) $x^2 - 10x - 25 = 0$

Answer Key:

1) $x = \{-14,8\}$ ***2)*** $x = \{4,16\}$ ***3)*** $x = 5 \pm 4\sqrt{2}$ ***4)*** $x = -4 \pm 2\sqrt{2}$

5) $x = 1 \pm 2\sqrt{6}$ ***6)*** $x = -3 \pm 6\sqrt{2}$ ***7)*** $x = -10 \pm 6\sqrt{3}$ ***8)*** $x = 12 \pm 8\sqrt{2}$

9) $x = \{-4,22\}$ ***10)*** $x = -6 \pm 5\sqrt{3}$ ***11)*** $x = \{-10,6\}$ ***12)*** $x = 7 \pm 4\sqrt{2}$

13) $x = 5 \pm 3\sqrt{2}$ ***14)*** $x = \{-5,-3\}$ ***15)*** $x = \{-8,10\}$ ***16)*** $x = -3 \pm 4\sqrt{3}$

Radical Equations

If you have the variable (usually x) under the radical, we cannot solve for it until we get it out from under the radical. The way to do this is to get the expression with the radical by itself and then square both sides. When the radical is by itself, squaring it gets rid of the radical. At this level, we will wind up with either a linear equation or a quadratic equation (both of which we covered in this packet).

Example 1: $7\sqrt{11x+3} - 46 = -4$

$+46 \quad +46$

$\frac{7\sqrt{11x+3}}{7} = \frac{42}{7}$

$\sqrt{11x+3} = 6$ *(Radical is isolated, square both sides.. this gets rid of the radical on the left side of the equation).*

$\left(\sqrt{11x+3}\right)^2 = (6)^2$

$11x + 3 = 36$

$-3 \quad -3$

$\frac{11x}{11} = \frac{33}{11}$

$x = 3$ *You MUST do your check to make sure your solution is not an extraneous root.

Check: $7\sqrt{11x+3} - 46 = -4 \quad x = 3$

$7\sqrt{11(3)+3} - 46 = -4$

$7\sqrt{33+3} - 46 = -4$

$7\sqrt{36} - 46 = -4$

$7(6) - 46 = -4$

$42 - 46 = -4$

$-4 = -4$ ✓

$x = \{3\}$

Example 2: $\sqrt{13x+30} - x = 0$

$+x \quad +x$

$\sqrt{13x+30} = x$

$(\sqrt{13x+30})^2 = (x)^2$ ***(Radical is isolated, square both sides)***

$13x + 30 = x^2$

$-13x \ -30 \qquad -13x - 30$

$0 = x^2 - 13x - 30$ *(Factor and solve)*

$0 = (x+2)(x-15)$

$x + 2 = 0 \qquad x - 15 = 0$

$-2 \quad -2 \qquad +15 \quad +15$

$x = -2 \qquad x = 15$ *(Check for extraneous roots)*

Check:

$\sqrt{13x+30} - x = 0$

X = -2

$\sqrt{13(-2)+30} - (-2) = 0$

$\sqrt{-26+30} - (-2) = 0$

$\sqrt{4} - (-2) = 0$

$2 - (-2) = 0$

$4 \neq 0$

x = 15

$\sqrt{13(15)+30} - (15) = 0$

$\sqrt{195+30} - (15) = 0$

$\sqrt{225} - (15) = 0$

$15 - (15) = 0$

$0 = 0$ ✓

$x = \{15\}$

Example 3: $\sqrt{12x - 20} - x = 1$

$+x \quad +x$

$\sqrt{12x - 20} = x + 1$ ***(Radical is isolated, square both sides)***

$(\sqrt{12x - 20})^2 = (x + 1)^2$

$(\sqrt{12x - 20})^2 = (x + 1)(x + 1)$

$12x - 20 = x^2 + 1x + 1x + 1$

$12x - 20 = x^2 + 2x + 1$

$-12x + 20 \qquad -12x + 20$

$0 = x^2 - 10x + 21$ *(Factor and solve)*

$0 = (x - 3)(x - 7)$

$x - 3 = 0 \quad x - 7 = 0$

$+3 \quad +3 \quad +7 \quad +7$

$x = 3 \quad x = 7$ *(Check for extraneous roots)*

Check:

$\sqrt{12x - 20} - x = 1$

X = 3

$\sqrt{12(3) - 20} - (3) = 1$

$\sqrt{36 - 20} - (3) = 1$

$\sqrt{16} - (3) = 1$

$4 - (3) = 1$

$1 = 1$ ✓

x = 7

$\sqrt{12(7) - 20} - (7) = 1$

$\sqrt{84 - 20} - (7) = 1$

$\sqrt{64} - (7) = 1$

$8 - (7) = 1$

$1 = 1$ ✓

$x = \{3,7\}$

Example 4: $\sqrt{11x + 36} + 6 = x$

$-6 \qquad -6$

$\sqrt{11x + 36} = x - 6$ ***(Radical is isolated, square both sides)***

$(\sqrt{11x + 36})^2 = (x - 6)^2$

$(\sqrt{11x + 36})^2 = (x - 6)(x - 6)$

$11x + 36 = x^2 - 6x - 6x + 36$

$11x + 36 = x^2 - 12x + 36$

$-11x - 36 \qquad -11x - 36$

$0 = x^2 - 23x$ *(Factor and solve)*

$0 = x(x - 23)$

$x = 0 \quad x - 23 = 0$

$+23 \quad +23$

$x = 23$

Check:

$\sqrt{11x + 36} + 6 = x$

X = 0

$\sqrt{11(0) + 36} + 6 = (0)$

$\sqrt{0 + 36} + 6 = 0$

$\sqrt{36} + 6 = 0$

$6 + 6 = 0$

12 $\neq$ *0* ✓

x = 7

$\sqrt{11(23) + 36} + 6 = (23)$

$\sqrt{253 + 36} + 6 = 23$

$\sqrt{289} + 6 = 23$

$17 + 6 = 23$

$23 = 23$ ✓

$x = \{23\}$

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1) $\sqrt{14x + 1} = 13$

2) $3\sqrt{8x + 9} - 24 = -3$

3) $\frac{1}{3}\sqrt{21 - 4x} + 17 = 20$

4) $\sqrt{72 - x} = x$

5) $\sqrt{-19x - 60} = x$

6) $-\sqrt{-19x - 60} = x$

7) $\sqrt{16x - 48} = x$

8) $\sqrt{12x + 4} - 3 = x$

9) $\sqrt{11x + 26} - x = 4$

10) $\sqrt{12x + 4} - 2 = x$

11) $\sqrt{8x + 1} - x = -1$

12) $\sqrt{41 - 2x}$ - 11 = x

13) $\sqrt{8x + 25}$ - x = -5

14) $\sqrt{-28 - 11x} = x$

15) $\sqrt{13x + 61} - 7 = x$

Q4 Quiz 4 Review:

1) $\sqrt{7x - 6} = x$ $x = \{1,6)$

2) $6\sqrt{6x - 2} - 55 = -7$ $x = 11$

3) $\frac{1}{2}\sqrt{11x + 31} + 9 = 16$ x = 15

4) $\sqrt{x^2 - 48} + 2 = x$ x = 13

5) $\sqrt{5x + 4} - x = -2$

x = {9}, 0 is an extraneous root

6) $\sqrt{14x - 6} - x = 3$

x = {3,5}

7) $\sqrt{10x - 1} + 4 = x$

x = {17},1 is an extraneous root

8) $\sqrt{9x + 1} - x = -1$

x = {11}, 0 is an extraneous root

9) $\frac{3}{4}\sqrt{13x - 38} - 8 = 1$ x = 14

10) $\sqrt{11x + 49} - 7 = x$ x = {-3,0}